Improvements to the method of dispersion relations for *B* nonleptonic decays

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We bring some clarifications and improvements to the method of dispersion relations in the external masses variables that we proposed recently for investigating the final state interactions in the B nonleptonic decays. We first present arguments for the existence of an additional term in the dispersion representation, which arises from an equal-time commutator in the Lehmann-Symanzik-Zimmerman formalism and can be approximated by the conventional factorized amplitude. The reality properties of the spectral function and the Goldberger-Treiman procedure to perform the hadronic unitarity sum are analyzed in more detail. We also improve the treatment of the strong interaction part by including the contributions of both t- and u-channel trajectories in the Regge amplitudes. Applications to the $B^0\to\pi^+\pi^-$ and $B^+\to\pi^0K^+$ decays are presented.

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I. INTRODUCTION

In a recent paper [1], we discussed rescattering effects in nonleptonic B decays into light pseudoscalar mesons, calculated by a method of dispersion relations in terms of the external masses. We recall that the analytic continuation in the external masses was originally investigated in the framework of axiomatic field theory [2]. In [1] we used an approach based on the Lehmann-Symanzik-Zimmermann (LSZ) reduction formalism [3], and we showed that the weak amplitude satisfies a dispersion representation in the mass squared of one final particle, with a spectral function given by the hadronic unitarity sum associated to rescattering effects. We mention that the analyticity domain in this heuristic treatment is much larger than the domain obtained by rigorous techniques. In the present work we discuss in more detail some aspects of the dispersion relations proposed in [1].

II. DISCUSSION OF THE METHOD

Defining the weak decay amplitude $A_{B\to P_1P_2}=A(m_B^2,m_1^2,m_2^2)$, where P_1,P_2 are light pseudoscalar mesons, we considered in [1] the LSZ reduction relation [3,4]

$$A(m_B^2,k_1^2,m_2^2)=\frac{i}{\sqrt{2\omega_1}}\int dx e^{ik_1x}\theta(x_0)\times\langle P_2(k_2)|[\eta_1(x),\mathcal{H}_w(0)]|B(p)\rangle, \tag{1}$$

where $\eta_1(x)=\mathcal{K}_x\phi_1(x)$ denotes the source of the meson P_1. This relation was the starting point for showing that the weak amplitude satisfies a dispersion representation in the mass variable k_1^2, with a discontinuity given by

$$\mathrm{disc}\,A(m_B^2,k_1^2,m_2^2)=\frac{1}{2\sqrt{2\omega_1}}\sum_n\delta(k_1+k_2-p_n)\times\langle P_2(k_2)|\eta_1|n\rangle\langle n|\mathcal{H}_w|B(p)\rangle. \tag{2}$$

In each term of the sum (2) the first matrix element represents the amplitude of the strong transition from the intermediate state $|n\rangle$ to the final state P_1P_2, for an off-shell meson P_1 with the invariant mass squared k_1^2, multiplied by the amplitude of the weak transition of B into the same intermediate state. Therefore, the discontinuity (2) describes the final state rescattering effects in the decay $B\to P_1P_2$. Note that all the particles involved in the weak transition are real, on-shell particles.

In Ref. [1] we obtained the whole amplitude from its discontinuity by means of a dispersion relation without subtractions. We note however that the presence of an additional term in the dispersion relations is not excluded, even if the subtractions in the dispersion integral are not necessary. To see the origin of such additional terms we notice that in deriving the relation (1) we retained only the contribution given by the action of the Klein-Gordon operator $\mathcal{K}_x$ on the interpolating field $\phi_1(x)$, since this was of interest for the analytic continuation. However, in the LSZ formula there are some additional terms produced by the action of $\mathcal{K}_x$ upon the function $\theta(x_0)$. These terms can be written as a sum of equal time commutators in the form [4]

$$\frac{i}{\sqrt{2\omega_1}}\int dx e^{ik_1x}\delta(x_0)\langle P_2(k_2)|-ik_{10}[\phi_1(x),\mathcal{H}_w(0)]+[\partial_0\phi_1(x),\mathcal{H}_w(0)]|B(p)\rangle. \tag{3}$$

As discussed in [4], these terms can contribute only if there is a ''direct'' connection between the interpolating field $\phi_1(x)$ and the operator $\mathcal{H}_w(0)$. Let us suppose that we applied the LSZ reduction formula to the final meson which does not contain the spectator quark in the B decay. Then we can assume that one of the currents entering the expression of $\mathcal{H}_w(0)$, more exactly the current which contains the fields of the quarks going into the final meson P_1, is related to the interpolating field by $j_\mu^{(1)}\approx\partial_\mu\phi_1(x)$. For the equal-time commutators we apply the canonical rules, which are satisfied, up to a renormalization constant, by the interpolating fields too [4]. Then the only nonzero term in (3) is given by the commutator

$$\delta(x_0)[\phi_1(x),\mathcal{H}_w(0)]=\delta(x_0)[\phi_1(x),\partial_0\phi_1(0)]\frac{\delta\mathcal{H}_w}{\delta\partial_0\phi_1}\approx\delta(x)j_0^{(2)}, \tag{4}$$

where $j_\mu^{(2)}$ is the second current in the weak Hamiltonian [for some terms of $\mathcal{H}_w(0)$ the argument applies after a Fierz rearrangement]. By including this expression in Eq. (3), identifying the normalization constant with the meson decay constant f_{P_1}, and restoring the Lorenz covariance, we notice that the additional term in the dispersion relation can be written in the form

$$if_{P_1}k_{1\mu}\langle P_2(k_2)|j_\mu^{(2)}|B(p)\rangle, \tag{5}$$

which is the conventional factorization in terms of a form factor and a meson decay constant. Of course, nonfactorizable corrections might arise if one goes beyond the simple relation, used in deriving Eq. (5), between one of the currents in the effective weak Hamiltonian and the interpolating field of a final meson.

By combining the new term (5) with the dispersive integral obtained from Eq. (1), we propose the following dispersion representation for the weak amplitude:

$$A(m_B^2,m_1^2,m_2^2)=A^{(0)}(m_B^2,m_1^2,m_2^2)+\frac{1}{\pi}\int_0^{(m_B-m_2)^2}dz\frac{\operatorname{disc}A(m_B^2,z,m_2^2)}{z-m_1^2-i\epsilon}. \tag{6}$$

As discussed above, the first term can be evaluated approximately using conventional factorization, while in the second one the dispersion variable is the mass of the meson which does not contain the spectator quark. The representation (6) gives, when the final state interactions are switched off, a term which can be approximated by the factorized amplitude (with possible hard scattering corrections), which is a reasonable consistency condition.[1] We are aware that we have not given a rigorous proof for the dispersion relation (6), but brought some general arguments, supported by the consistency of the physical picture.

Below, we shall investigate in more detail the evaluation of the above dispersion relation. Our first remark is that in the unitarity sum (2) one can use as a complete set of hadronic states $|n\rangle$ either the ''in'' or the ''out'' states. The equivalence of these two sets was used in [5] to prove the reality of the spectral function for T (or CP) conserving interactions. Let us consider what happens if the weak Hamiltonian contains a CP violating part. In the standard model the weak hamiltonian $\mathcal{H}_w$ has the general form

$$\mathcal{H}_w=\mathcal{O}_1+\mathcal{O}_2e^{i\gamma}+\text{H.c.}, \tag{7}$$

where $\mathcal{O}_j$, $j=1,\ 2$, are products of vector and axial vector weak currents involving only real coefficients, and γ is the weak angle of the Cabibbo-Kobayashi-Maskawa (CKM) matrix in the standard parametrization [$\gamma=\arg(V_{ub}^*)$]. Then the spectral function defined in Eq. (2) can be written as

$$\operatorname{disc}A(m_B^2,z,m_2^2)=\sigma_1(z)+\sigma_2(z)e^{i\gamma}, \tag{8}$$

where $\sigma_1(z)[\sigma_2(z)]$ are obtained by replacing $\mathcal{H}_w$ in the right-hand side of Eq. (2) with $\mathcal{O}_1$ $[\mathcal{O}_2]$, respectively (we took for convenience a process involving the weak phase γ). It is easy to show that $\sigma_1(z)$ and $\sigma_2(z)$ are real functions. Indeed, let us assume that a complete set $|n,\text{in}\rangle$ is inserted in the unitarity sum of Eq. (2). Following [5], we can express the two matrix elements in this sum as

$$\langle P_2(k_2)|\eta_1|n,\text{in}\rangle=\langle P_2(k_2)|(PT)^{-1}(PT)\eta_1(PT)^{-1}(PT)|n,\text{in}\rangle=\langle P_2(k_2)|\eta_1|n,\text{out}\rangle^* \tag{9}$$

and

$$\langle n,\text{in}|\mathcal{O}_j|B(p)\rangle=\langle n,\text{in}|(PT)^{-1}(PT)\mathcal{O}_j(PT)^{-1}(PT)|B(p)\rangle=\langle n,\text{out}|\mathcal{O}_j|B(p)\rangle^*. \tag{10}$$

We used here the transformation properties of the $\mathcal{O}_j$ operators under P and T transformations, and the fact that under space-time reversal the particles conserve their momenta, the in (out) states becoming out (in) states, respectively. Moreover, the matrix elements are replaced by their complex conjugates, given the antiunitary character of the operator T. By using the relations (9) and (10) in Eq. (2) we obtain

$$\begin{aligned}\sigma_j(z)&=\frac{1}{2\sqrt{2\omega_1}}\sum_n\ \delta(k_1+k_2-p_n)\langle P_2(k_2)|\eta_1|n,\text{in}\rangle\langle n,\text{in}|\mathcal{O}_j|B(p)\rangle\\&=\frac{1}{2\sqrt{2\omega_1}}\left[\sum_n\ \delta(k_1+k_2-p_n)\langle P_2(k_2)|\eta_1|n,\text{out}\rangle\langle n,\text{out}|\mathcal{O}_j|B(p)\rangle\right]^*=\sigma_j^*(z),\quad j=1,2,\end{aligned} \tag{11}$$

[1]We thank M. Beneke for emphasizing this point.

where the equivalence between the complete sets of in and out states in the definition of $\sigma(z)$ is taken into account. Equations (8) and (11) express in a detailed form the reality properties of the discontinuity, and bring a correction to the relation (16) given in Ref. [1].

From Eq. (11) it follows that the discontinuities $\sigma_j(z)$ are manifestly real only if the intermediate states form a complete set. If the unitarity sum is truncated, this property is lost, since various terms have complex phases which do not compensate each other in an obvious way. By inserting in Eq. (2) a set of states $|n,\text{out}\rangle$, we obtain for each term the product of the weak amplitudes with the complex conjugates of the strong amplitudes. If the set which is inserted consists of $|n,\text{in}\rangle$ states, then in Eq. (2) the strong amplitudes appear as such, while in the weak amplitudes we must take the complex conjugate of the strong phases [the weak phase multiplying the operator $\mathcal{O}_2$ in Eq. (7) is unmodified, since the same part of the weak Hamiltonian acts on both in and out states].

As noticed in Ref. [5], it is convenient to write the complete set of states $|n\rangle$ as a combination $1/2|n,\text{in}\rangle + 1/2|n,\text{out}\rangle$. In the case of CP conserving interactions this procedure maintains the reality condition of the spectral function (this method was used in the so-called Omnès solution for the electromagnetic form factor [6]). In our case it is easy to show that the Goldberger-Treiman procedure respects at all stages of approximation the specific reality conditions expressed in the relations (8) and (11).

We consider now the two-particle approximation, when the dispersion relation takes a very simple form. Indeed, in this case the on-shell weak decay amplitudes $A_{B\to P_3P_4}$ appearing in the unitarity sum (2) are independent of the phase space integration variables, and also of the dispersion variable z. Therefore, the dispersion representation (6) becomes an algebraic relation among on-shell weak amplitudes [1].

If we insert in Eq. (2) a set of states $|P_3P_4,\text{out}\rangle$, we obtain

$$\text{disc}\, A_{B\to P_1P_2} = \sum_{\{P_3P_4\}} C^*_{P_3P_4;P_1P_2}(z) A_{B\to P_3P_4}, \quad (12)$$

where $C^*_{P_3P_4;P_1P_2}(z)$ are the complex conjugates of the coefficients

$$C_{P_3P_4;P_1P_2}(z) = \frac{1}{2}\int \frac{d^3\mathbf{k}_3}{(2\pi)^3 2\omega_3}\frac{d^3\mathbf{k}_4}{(2\pi)^3 2\omega_4}(2\pi)^4\delta^{(4)} \times (p-k_3-k_4)\mathcal{M}_{P_3P_4\to P_1P_2}(s,t), \quad (13)$$

defined in terms of the strong amplitudes $\mathcal{M}_{P_3P_4\to P_1P_2}(s,t)$, where s, t, and u are the Mandelstam variables. These coefficients depend on the masses of all the particles participating in the rescattering process, in particular, they depend on the dispersive variable $z = k_1^2$.

Similarly, by including in Eq. (2) a set of states $|P_3P_4, \text{in}\rangle$ we obtain

$$\text{disc}\, A_{B\to P_1P_2} = \sum_{\{P_3P_4\}} C_{P_3P_4;P_1P_2}(z)\overline{A}_{B\to P_3P_4}, \quad (14)$$

where, according to the above discussion, the amplitude $\overline{A}_{B\to P_3P_4}$ is obtained from $A_{B\to P_3P_4}$ by changing the sign of the strong phase, namely,

$$\overline{A}_{B\to P_3P_4} = |A_{B\to P_3P_4}|e^{-i\phi_s}e^{i\phi_w},$$

where $\phi_s(\phi_w)$ denotes the strong (weak) phase.

Now by performing the symmetric Goldberger-Treiman summation as explained above, we obtain, instead of Eq. (12) or Eq. (14), the expression

$$\text{disc}\, A_{B\to P_1P_2} = \frac{1}{2}\sum_{\{P_3P_4\}} C_{P_3P_4;P_1P_2}(z)\overline{A}_{B\to P_3P_4} + \frac{1}{2}\sum_{\{P_3P_4\}} C^*_{P_3P_4;P_1P_2}(z)A_{B\to P_3P_4}. \quad (15)$$

With this discontinuity, the dispersion relation (6) becomes

$$A_{B\to P_1P_2} = A^{(0)}_{B\to P_1P_2} + \frac{1}{2}\sum_{\{P_3P_4\}} \Gamma_{P_3P_4;P_1P_2}\overline{A}_{B\to P_3P_4} + \frac{1}{2}\sum_{\{P_3P_4\}} \overline{\Gamma}_{P_3P_4;P_1P_2}A_{B\to P_3P_4}, \quad (16)$$

where $A^{(0)}_{B\to P_1P_2}$ can be approximated by the amplitude in the factorization limit, and the coefficients $\Gamma_{P_3P_4;P_1P_2}$ and $\overline{\Gamma}_{P_3P_4;P_1P_2}$ are defined as

$$\Gamma_{P_3P_4;P_1P_2} = \frac{1}{\pi}\int_0^{(m_B-m_2)^2} dz \frac{C_{P_3P_4;P_1P_2}(z)}{z-m_1^2-i\epsilon} \quad (17)$$

and

$$\overline{\Gamma}_{P_3P_4;P_1P_2} = \frac{1}{\pi}\int_0^{(m_B-m_2)^2} dz \frac{C^*_{P_3P_4;P_1P_2}(z)}{z-m_1^2-i\epsilon}. \quad (18)$$

Equations (15)–(18) are the result of the Goldberger-Treiman procedure in the presence of CP violating interactions, replacing the corresponding relations (38) and (39) given in [1].

The strong amplitudes $\mathcal{M}_{P_3P_4;P_1P_2}(s,t)$ entering the expression (13) of the coefficients $C_{P_3P_4;P_1P_2}(z)$ are evaluated at the c.m. energy squared $s = m_B^2$, which, as emphasized in [7], is high enough to justify the application of Regge theory [8]. The amplitudes $\mathcal{M}_{P_3P_4;P_1P_2}(s,t)$ can be expressed therefore as sums over Regge exchanges in the crossed channels, more exactly [9]: near the forward direction (small t) the t-channel exchanges are taken into account, while near the backward direction (small u) the u-channel exchanges are considered. The standard form of a Regge amplitude given by a trajectory exchanged in the t channel is

$$-\gamma(t)\frac{\tau+e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}\left(\frac{s}{s_o}\right)^{\alpha(t)}, \tag{19}$$

where $\gamma(t)$ is the residue function, τ the signature, $\alpha(t)=\alpha_0+\alpha' t$ the linear trajectory, and $s_0\approx 1$ GeV^2. A Regge trajectory exchanged in the u channel gives an expression similar to Eq. (19), with t replaced by u.

Using the signatures $\tau=1$ for $C=1$ trajectories and $\tau=-1$ for $C=-1$, we express the strong amplitude near the foward direction as [1]

$$\mathcal{M}_{P_3P_4;P_1P_2}(s,t)=-\sum_{V=P,f,A_2,K^{**}\ldots}\gamma^V_{P_3P_4;P_1P_2}(t)\frac{e^{-i\pi\alpha_V(t)/2}}{\sin(\pi\alpha_V(t)/2)}\left(\frac{s}{s_0}\right)^{\alpha_V(t)}+\sum_{V=\rho,K^*\ldots}i\gamma^V_{P_3P_4;P_1P_2}(t)\frac{e^{-i\pi\alpha_V(t)/2}}{\cos(\pi\alpha_V(t)/2)}\left(\frac{s}{s_0}\right)^{\alpha_V(t)}, \tag{20}$$

where the sum extends over the t-channel poles. The first sum includes the Pomeron (which contributes only to the elastic scattering) and tensor particles, the second sum includes vector particles. In Ref. [1] we used this expression in the integral over the phase space in Eq. (13), which finally reduces to an integral over the c.m. scattering angle θ. Since the Regge amplitudes (20) decrease exponentially at large t, the dominant contribution in the integral is brought by the forward region. This gives a correct result for amplitudes which are small near the backward direction (for example, in processes where the u channel is exotic), otherwise, it misses in general the contribution due to large angles. As discussed in [9], it is more appropriate to separate the integration over the scattering angle θ in two regions, one for small angles using the Regge expression (20), the other for large angles where a similar expression

$$\mathcal{M}_{P_3P_4;P_1P_2}(s,t)=-\sum_{V=f,A_2,K^{**}\ldots}\gamma^V_{P_3P_4;P_1P_2}(u)\frac{e^{-i\pi\alpha_V(u)/2}}{\sin(\pi\alpha_V(u)/2)}\left(\frac{s}{s_0}\right)^{\alpha_V(u)}+\sum_{V=\rho,K^*\ldots}i\gamma^V_{P_3P_4;P_1P_2}(u)\frac{e^{-i\pi\alpha_V(u)/2}}{\cos(\pi\alpha_V(u)/2)}\left(\frac{s}{s_0}\right)^{\alpha_V(u)} \tag{21}$$

given by the u-channel Regge trajectories is valid. Following [9], in performing the unitarity integral (13) we adopt the expression (20) of the amplitude $\mathcal{M}_{P_3P_4;P_1P_2}(s,t)$ for $\cos\theta>0$ and the expression (21) for $\cos\theta<0$. Assuming the residua functions $\gamma^V_{P_3P_4;P_1P_2}(t)$ and $\gamma^V_{P_3P_4;P_1P_2}(u)$ to be constant along the relevant integration ranges, and neglecting also the $t(u)$ dependence of the denominators in Eqs. (20) and (21), the integration over the momenta $\mathbf{k}_3$ and $\mathbf{k}_4$ in Eq. (13) is straightforward, knowing the kinematic relations between the Mandelstam variables t and u and the scattering angle. The coefficients $C_{P_3P_4;P_1P_2}$ can be expressed as

$$C_{P_3P_4;P_1P_2}(z)=\sum_{\{V_t\}}\xi_{V_t}\gamma^{V_t}_{P_3P_4;P_1P_2}\kappa^{V_t}_{P_3P_4;P_1P_2}(z)+\sum_{\{V_u\}}\xi_{V_u}\gamma^{V_u}_{P_3P_4;P_1P_2}\kappa^{V_u}_{P_3P_4;P_1P_2}(z), \tag{22}$$

where the first (second) sum includes the contribution of the $t(u)$-channel trajectories. In Eq. (22), ξ_V is a numerical factor due to the signature (equal to -1 for the Pomeron, $i\sqrt{2}$ for $C=-1$ trajectories, and $-\sqrt{2}$ for $C=1$ physical trajectories). The coefficients $\kappa^{V_t}_{P_3P_4;P_1P_2}(z)$ appearing in the first sum have the expression

$$\kappa^{V_t}_{P_3P_4;P_1P_2}(z)=\frac{k_{34}}{16\pi m_B}\mathcal{R}^{-1}_{V_t}(z)[e^{\mathcal{R}_{V_t}(z)}-1]\exp\left[[\alpha_{0,V_t}+\alpha'_{V_t}t_0(z)]\left(\ln\frac{m_B^2}{s_0}-i\frac{\pi}{2}\right)\right], \tag{23}$$

obtained by integration over the region $0<\cos\theta<1$ of the phase space. We used the notation [1]

$$\mathcal{R}_V(z)=2\alpha'_V k_{12}(z)k_{34}\left(\ln\frac{m_B^2}{s_0}-i\frac{\pi}{2}\right), \tag{24}$$

where k_{12} and k_{34} denote the c.m. three momenta and

$$t_0(z)=z+m_3^2-\frac{(m_B^2+m_3^2-m_4^2)(m_B^2+z-m_2^2)}{2m_B^2}. \tag{25}$$

As for the coefficients $\kappa^{V_u}_{P_3P_4;P_1P_2}(z)$ appearing in the second sum of Eq. (22), they are given by the region $-1<\cos\theta<0$ of the phase space integral, and their expression is similar to Eq. (23), with the t-channel trajectories replaced by the u-channel trajectories, and $t_0(z)$ replaced by the variable $u_0(z)$, which is obtained from Eq. (25) by interchanging the places of m_3 and m_4. The new relations (22)–(23) improve the corresponding equations (28)–(30) given in Ref. [1].

III. APPLICATIONS TO THE $B^0\to\pi^+\pi^-$ AND $B^+\to\pi^0K^+$ DECAYS

An application of the dispersive formalism to the decay $B^0\to\pi^+\pi^-$ was already discussed in Ref. [1]. In the present work, we reconsider this analysis using the improvements presented above. As intermediate states $\{P_3P_4\}$ in the dispersion relation (16) we include $\pi^+\pi^-$ for the elastic rescattering, and two-particle states responsible for the soft inelastic rescattering. We take into account the contribution of the lowest pseudoscalar mesons: $\pi^0\pi^0$, K^+K^-, $K^0\overline{K^0}$, $\eta_8\eta_8$, $\eta_1\eta_1$, and $\eta_1\eta_8$. Here η_8 and η_1 denote the SU(3) octet and singlet, respectively, and we assumed for simplicity that the mixing is negligible (we mention that the singlet η_1 was not included in the previous analysis [1]). Then, assuming SU(3) flavor symmetry, all the B decay amplitudes entering the dispersion relation can be expressed in terms of a certain set of amplitudes associated to quark diagrams [10]. Following [10], we shall assume that the annihilation, penguin annihilation, electroweak penguin, and exchange diagrams are negligible. For simplicity, we also neglect in this first step the tree color suppressed amplitude, as in Ref. [1]. Of course, the final state interactions can modify the naive estimates based on quark diagrams [11], therefore, the analysis presented below is only a first approximation to be refined in a future work. With these assumptions $A_{B^0\to K^+K^-}=0$, and the remaining amplitudes entering the dispersion relation have the expressions

$$
\begin{aligned}
A_{B^0\to\pi^+\pi^-}&=-(A_Te^{i\gamma}+A_Pe^{-i\beta}),\\
A_{B^0\to\pi^0\pi^0}&=\frac{1}{\sqrt{2}}A_Pe^{-i\beta},\\
A_{B^0\to K^0\overline{K^0}}&=A_Pe^{-i\beta},\\
A_{B^0\to\eta_8\eta_8}&=\frac{1}{3\sqrt{2}}A_Pe^{-i\beta},\\
A_{B^0\to\eta_1\eta_1}&=\sqrt{\frac{2}{3}}A_Pe^{-i\beta},\\
A_{B^0\to\eta_8\eta_1}&=-\sqrt{\frac{2}{3}}A_Pe^{-i\beta},
\end{aligned}
\tag{26}
$$

the weak phases being defined as $\beta=\arg(V_{td}^*)$ and $\gamma=\arg(V_{ub}^*)$.

The determination of the Regge residua $\gamma^V_{P_3P_4;P_1P_2}$ which appear in expression (22) of the coefficients $C_{P_3P_4;P_1;P_2}$ was described in detail in [1]. Using the optical theorem and the usual Regge parametrization of the total hadronic cross sections we obtain the following values

$$
\begin{aligned}
\gamma^P_{\pi^+\pi^-;\pi^+\pi^-}&\equiv\gamma_P^2=25.6,\\
\gamma^\rho_{\pi^+\pi^-;\pi^+\pi^-}&\equiv\gamma_1^2=31.4,\\
\gamma^{f_8}_{\pi^+\pi^-;\pi^+\pi^-}&\equiv\gamma_2^2=35.3,
\end{aligned}
\tag{27}
$$

for the residua of the Pomeron, the ρ and the f mesons, respectively. All the other residua are expressed in terms of the couplings γ_1^2 and γ_2^2 by flavor SU(3) symmetry. For completeness we give their values in Table I, where we indicate for each trajectory the result given by the Clebsch-Gordan coefficients, the sign due to the relation between the quark content of the mesons and the SU(3) vector assigned to them (with the convention of Ref. [10]), and the Regge factor ξ_V defined below Eq. (22). The quantity $\xi_V\,\gamma^V_{P_3P_4;P_1P_2}$ appearing in Eq. (22) is obtained for each trajectory by taking the product of the values in the last three columns of Table I.

As follows from Table I, the process $\pi^0\pi^0\to\pi^+\pi^-$ actually does not contribute to the dispersion relation, since the contribution of the ρ trajectory in the t channel is exactly compensated in the unitarity integral by an equal term given by the u channel. On the contrary, in the case of the A_2 trajectory, the contributions in the t and u channels are equal and add to each other. We also note that the couplings of the singlet η_1 given in Table I are obtained by assuming an exact U(3) symmetry. We took into account in the numerical calculations that deviations from these values are possible due to the U(1) anomaly.

With the input described above one can calculate easily the coefficients (22) and the dispersive integrals (17),(18) giving the coefficients Γ and $\overline{\Gamma}$. By inserting these coefficients and the explicit expressions (26) of the decay amplitudes in the dispersion relation (16), we derive an algebraic equation involving the complex quantities A_T and A_P. Let us denote $R=|A_P/A_T|$ and $\delta=\delta_P-\delta_T$, where $\delta_T(\delta_P)$ is the strong phase of $A_T(A_P)$, respectively. Then, after dividing both sides of the relation (16) by $-|A_T|$ and multiplying by $e^{-i\delta_T}$, we obtain the following equation:

$$
\begin{aligned}
e^{i\gamma}+Re^{i\delta}e^{-i\beta}=\frac{e^{-i\delta_T}}{|A_T|}[A_T^{(0)}e^{i\gamma}+A_P^{(0)}e^{-i\beta}]-[(0.01+1.27\,i)+(0.75-1.01\,i)e^{-2i\delta_T}]e^{i\gamma}\\
+R[-(1.97+2.64\,i)e^{i\delta}e^{-i\beta}-(1.78-1.99i)e^{-i\delta}e^{-i\beta}e^{-2i\delta_T}].
\end{aligned}
\tag{28}
$$

TABLE I. Values of the Regge residua of the rescattering amplitudes in $B^0 \to \pi^+\pi^-$: column II indicates the channel, III the Regge trajectories, IV the coupling given by SU(3) Clebsch-Gordan coefficients, V the additional sign due to the definition of the meson states, and VI the Regge factor ξ_V.

I	II	III	IV	V	VI
$\pi^+\pi^- \to \pi^+\pi^-$					
	t	ρ	γ_1^2	+	$i\sqrt{2}$
	t	f_8	γ_2^2	+	$-\sqrt{2}$
	u	exotic			
$\pi^0\pi^0 \to \pi^+\pi^-$					
	t	ρ	γ_1^2	−	$i\sqrt{2}$
	u	ρ	$-\gamma_1^2$	−	$i\sqrt{2}$
$K^0\overline{K}^0 \to \pi^+\pi^-$					
	t	exotic			
	u	K^*	$-1/2\gamma_1^2$	−	$i\sqrt{2}$
	u	K^{**}	$3/2\gamma_2^2$	−	$-\sqrt{2}$
$\eta_8\eta_8 \to \pi^+\pi^-$					
	t	A_2	γ_2^2	−	$-\sqrt{2}$
	u	A_2	γ_2^2	−	$-\sqrt{2}$
$\eta_1\eta_1 \to \pi^+\pi^-$					
	t	A_2	$5\gamma_2^2$	−	$-\sqrt{2}$
	u	A_2	$5\gamma_2^2$	−	$-\sqrt{2}$
$\eta_8\eta_1 \to \pi^+\pi^-$					
	t	A_2	$\sqrt{5}\gamma_2^2$	−	$-\sqrt{2}$
	u	A_2	$\sqrt{5}\gamma_2^2$	−	$-\sqrt{2}$

where $A_T^{(0)}$ and $A_P^{(0)}$ are the tree and penguin amplitudes in the factorization approximation. We mention that in Ref. [1] the equation similar to Eq. (28) did not contain the factorized amplitude and, due to an improper application of the Goldberger-Treiman technique, the weak phase β appearing in the last term had a wrong sign.

Multiplying both sides of Eq. (28) by $e^{i\beta}$ one notices that the weak angles appear in the combination $\gamma+\beta=\pi-\alpha$, where α is the third angle of the unitarity triangle. Then, solving the complex equation for R and α we derive the expressions

$$R(\delta_T,\delta)=\left|\frac{1.010+1.27\,i+(0.75-1.01\,i)e^{-2i\delta_T}-(e^{-i\delta_T}/A_T)[A_T^{(0)}+A_P^{(0)}e^{-i(\gamma+\beta)}]}{-(1.97+2.64\,i)e^{i\delta}-(1.78-1.99i)e^{-i\delta}e^{-2i\delta_T}}\right|, \tag{29}$$

and

$$\alpha(\delta_T,\delta)=\pi+\arg\left[1.01+1.27\,i+(0.75-1.01\,i)e^{-2i\delta_T}-\frac{A_T^{(0)}}{|A_T|}e^{-i\delta_T}\right]-\arg\left[-(1.97+2.64\,i)e^{i\delta}-(1.78-1.99i)e^{-i\delta}e^{-2i\delta_T}+\frac{e^{-i\delta_T}}{R}\frac{A_P^{(0)}}{|A_T|}\right], \tag{30}$$

where in the last equation we use R from Eq. (29). The evaluation of these expressions requires the knowledge of the ratios $A_P^{(0)}/A_T^{(0)}$ and $A_T^{(0)}/A_T$. We use for illustration $A_P^{(0)}/A_T^{(0)}=0.08$ [12], and a reasonable choice $A_T^{(0)}/A_T\approx 0.9$. The expression of R contains also the weak angle $\beta+\gamma$, but the dependence on this parameter is very weak. In Fig. 1 we represent R [Eq. (29)] as a function of the phase difference δ, for two values of δ_T. We recall that the ratio R is expected to be less than one, and such values are obtained for both $\delta_T=0$ and $\delta_T=\pi/12$. The ratio R is actually a periodic function of δ with a period equal to π, which implies that discrete ambiguities affect the determination of this phase difference for a given value of R.

In Fig. 2 we show α as a function of δ, for $\delta_T=0$ and $\delta_T=\pi/12$. Since one expects positive values of α, the curves shown in Fig. 2 indicate that negative values of δ are preferred. We have checked that the results are rather stable with respect to the variation of the η_1 couplings. For instance, by varying the corresponding Regge residua from 0 up to a value 2 or 3 times larger than the value given in

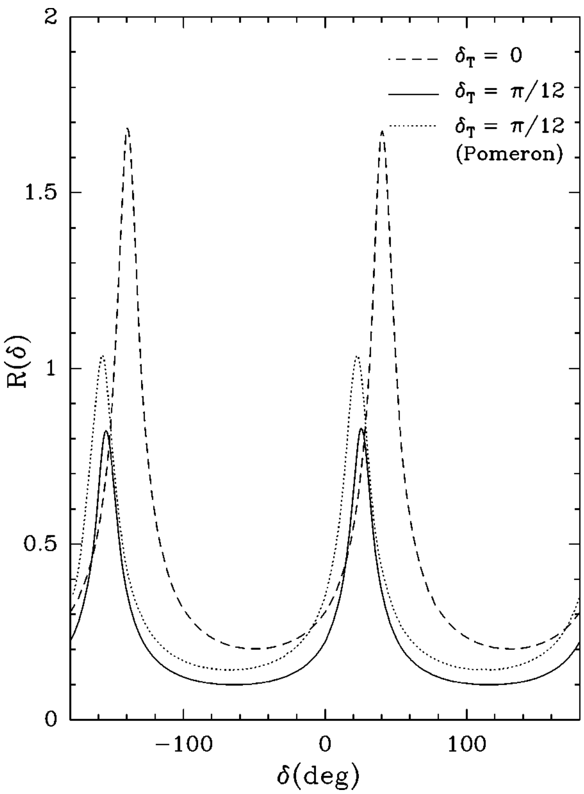

FIG. 1. The ratio $R=|A_P/A_T|$ given by Eq. (29) as a function of the strong phase difference δ, solid curve $\delta_T=\pi/12$, dashed curve $\delta_T=0$. The dotted curve is obtained for $\delta_T=\pi/12$, by keeping only the contribution of the Pomeron in the Regge amplitudes.

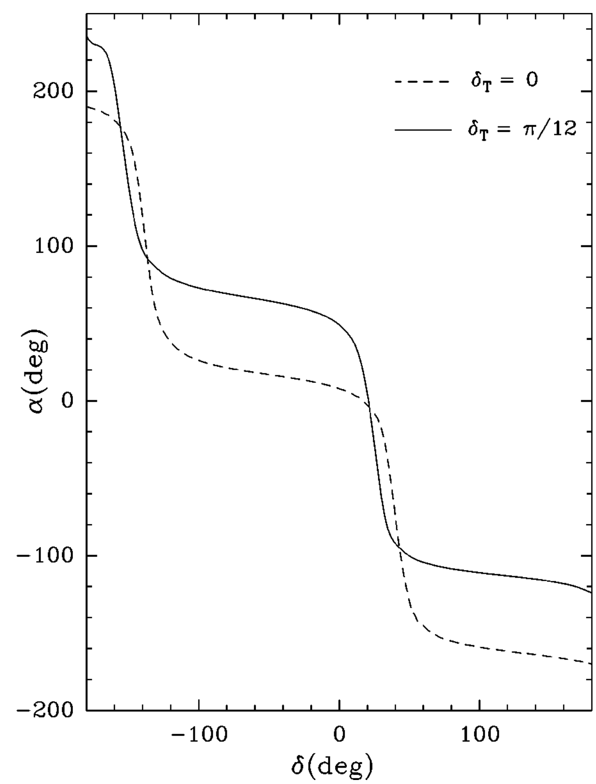

FIG. 2. The weak phase α [Eq. (30)] as a function of the strong phase difference δ, solid curve $\delta_T=\pi/12$, and dashed curve $\delta_T=0$.

Table I, we notice that the curves in Figs. 1 and 2 are slightly shifted, but the general behavior remains the same. Actually, the dominant contribution is given by the elastic channel, more precisely by the Pomeron, as is seen in Fig. 1, where we show the ratio R for $\delta_T=\pi/12$, keeping only the contribution of the Pomeron in the Regge amplitudes (dotted curve).

For comparison, in Ref. [1] the corresponding equation did not contain the amplitude in the factorization limit, and instead of the angles $\beta+\gamma$ and δ, we had the angles γ and $\delta-\beta$, respectively [see the remark below Eq. (28)]. Figures 1 and 2 given in [1] become therefore meaningful if the arguments are replaced accordingly; they represent in fact R and δ as functions of $\beta+\gamma=\pi-\alpha$, neglecting the factorized term. The values of R obtained for $\delta_T=0$ were larger than one, suggesting that small values of δ_T are excluded. Now, as seen in Fig. 1, the improved dispersion representation proposed in the present work indicates that reasonable values of R are compatible with small values of δ_T.

In a second application we consider the decay $B^+\to\pi^0K^+$, taking as intermediate states P_3P_4 the pseudoscalar mesons π^0K^+, π^+K^0, η_8K^+, and η_1K^+, allowed by the strong interactions. A model independent analysis based on isospin symmetry done in Ref. [13] gives for the amplitude of $B^+\to\pi^0K^+$ decay the expression

$$A_{B^+\to\pi^0K^+}=-\frac{P}{\sqrt{2}}[1-\epsilon_a e^{i\gamma}e^{i\eta}-\epsilon_{3/2}e^{i\phi}(e^{i\gamma}-\delta_{EW})], \tag{31}$$

where P denotes the dominant penguin amplitude, δ_{EW} is an electroweak correction, and ϵ_a, $\epsilon_{3/2}$, η and ϕ are hadronic parameters ($\phi=\phi_{3/2}-\phi_P$, where $\phi_{3/2}$ is the strong phase of the $I=3/2$ amplitude and ϕ_P the phase of P). According to [13], the term proportional to ϵ_a, which is due to nondominant penguin and annihilation topologies, is smaller than the last terms appearing in Eq. (31). Neglecting in a first approximation this term and using flavor SU(3) symmetry for the weak decays [10], we write the amplitudes which contribute to the dispersion relation as

$$A_{B^+\to\pi^0K^+}=-\frac{P}{\sqrt{2}}[1-re^{i\phi}],\quad A_{B^+\to\pi^+K^0}=P,$$

$$A_{B^+\to\eta_8K^+}=\frac{P}{\sqrt{6}}[1+re^{i\phi}],$$

$$A_{B^+\to\eta_1K^+}=-\frac{P}{\sqrt{3}}[2-re^{i\phi}], \tag{32}$$

where

$$r=\epsilon_{3/2}(e^{i\gamma}-\delta_{EW}). \tag{33}$$

We need also the amplitude $A^{(0)}_{B^+\to\pi^0K^+}$ in the factorized approximation, which we write as [14]

$$A^{(0)}_{B^+\to\pi^0K^+}\approx-\frac{P^{(0)}}{\sqrt{2}}[1-0.35e^{i\gamma}].$$

The Regge residua entering the coefficients (22) can be ex-

TABLE II. Values of the Regge residua for the rescattering amplitudes in the $B^+ \to \pi^0 K^+$ case. The meaning of the columns is the same as in Table I.

I	II	III	IV	V	VI
$\pi^0 K^+ \to \pi^0 K^+$					
	t	f_8	$1/2\gamma_2^2$	$-$	$-\sqrt{2}$
	u	K^*	$1/4\gamma_1^2$	$+$	$i\sqrt{2}$
	u	K^{**}	$3/4\gamma_2^2$	$+$	$-\sqrt{2}$
$\pi^+ K^0 \to \pi^0 K^+$					
	t	ρ	$-\sqrt{2}/2\gamma_1^2$	$+$	$i\sqrt{2}$
	u	K^*	$-\sqrt{2}/4\gamma_1^2$	$-$	$i\sqrt{2}$
	u	K^{**}	$-3\sqrt{2}/4\gamma_2^2$	$-$	$-\sqrt{2}$
$\eta_8 K^+ \to \pi^0 K^+$					
	t	A_2	$-\sqrt{3}/2\gamma_2^2$	$-$	$-\sqrt{2}$
	u	K^*	$\sqrt{3}/4\gamma_1^2$	$+$	$i\sqrt{2}$
	u	K^{**}	$-\sqrt{3}/4\gamma_2^2$	$+$	$-\sqrt{2}$
$\eta_1 K^+ \to \pi^0 K^+$					
	t	A_2	$-\sqrt{15}/2\gamma_2^2$	$-$	$-\sqrt{2}$
	u	K^{**}	$\sqrt{15}/2\gamma_2^2$	$+$	$-\sqrt{2}$

pressed in terms of the same parameters γ_P^2, γ_1^2, and γ_2^2 as defined in Eq. (27) by using SU(3). We fix the Pomeron coupling to the same value as in the $\pi\pi$ case, and take for the other trajectories the values listed in Table II, where the meaning of the columns is the same as in Table I.

By inserting the amplitudes (32) and the new coefficients Γ and $\bar{\Gamma}$, calculated as above, in the dispersion relation (16), we obtain after simplifying with $|P|$ a complex equation involving the parameters r, ϕ_P, and ϕ. From this equation we obtain for r the expression

$$r(\phi_P,\phi)=(0.12+0.09\,i)\frac{(5.48+4.05\,i)-(6.98-4.8\,i)e^{2i\phi_P}+i(-3.33+e^{i\gamma})(P^{(0)}/|P|)e^{i\phi_P}}{(0.16+0.75\,i)e^{-i\phi}-e^{i\phi}e^{2i\phi_P}}. \tag{34}$$

The expression of r contains also the ratio $P^{(0)}/|P|$ and the weak angle γ (actually the dependence of r on γ is very weak). In Fig. 3 we show for illustration the real and the imaginary parts of r as functions of ϕ, for $\phi_P=\pi/6$, using the estimate $P^{(0)}/|P|\approx 0.7$. From the definition (33), calculated with the parameters $\epsilon_{3/2}=0.24$ and $\delta_{EW}=0.64$ given in Ref. [13], we can extract the weak angle γ, defined as $\gamma=\arg(r/\epsilon_{3/2}+\delta_{EW})$. In Fig. 4 we show the variation of γ as a function of ϕ for $\phi_P=\pi/6$.

IV. CONCLUSIONS

In the present work we brought several improvements to the dispersion formalism proposed in Ref. [1] for investigating the hadronic parameters in B nonleptonic decays. The main modification consists in the discovery of an additional term in the dispersion representation. The new dispersion relation is given by Eq. (6), where the first term can be approximated by the factorized amplitude, and the dispersive variable is the mass of the final meson which does not contain the spectator quark. We mention that Eq. (6) is not a subtracted dispersion relation, the origin of the additional term being an equal time commutator in the LSZ formalism. We also treated more carefully the Goldberger-Treiman procedure to perform the unitarity sum, and refined the Regge model by the inclusion of both the t- and u-channel trajectories. We emphasize that the analytic continuation in the external mass could be kept under control in the present context, since the Regge dynamics is rather universal (as long as the masses are small with respect to the energy).

With the improvements mentioned above we reconsidered the analysis of the $B^0\to\pi^+\pi^-$ decay previously made, and discussed also the process $B^+\to\pi^0K^+$. In the present calculation we restricted the sum over intermediate states to the lowest pseudoscalar mesons, and invoked flavor symmetry to reduce the number of unknown amplitudes. The credibility of the results relies on the validity of the assumption that the higher states do not contribute significantly in the dispersion relation. There are several arguments in favor of this assumption. First, our results show that the dominant contribution is given by the Pomeron. Higher mass states are suppressed by

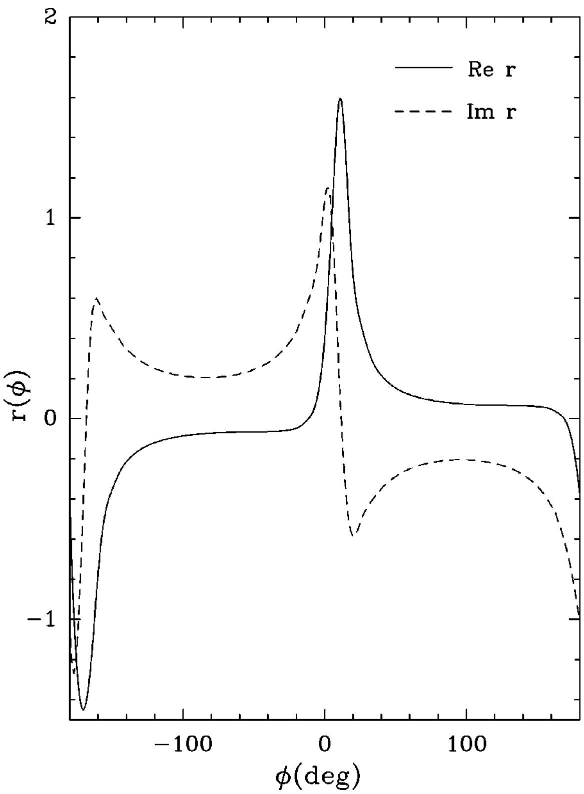

FIG. 3. The real part (solid curve) and the imaginary part (dashed curve) of r defined in Eqs. (33) and (34), as functions of the strong phase difference ϕ, for $\phi_P = \pi/6$.

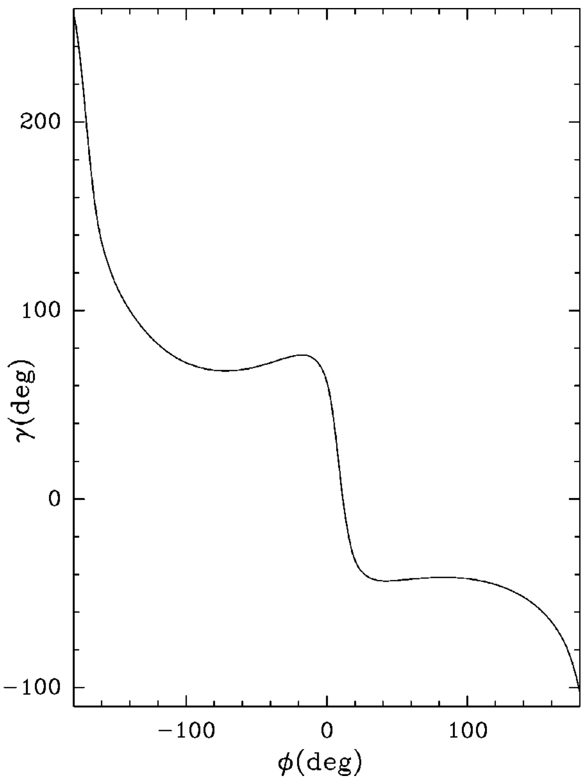

FIG. 4. The weak phase $\gamma = \arg(r/\epsilon_{3/2} + \delta_{EW})$, as a function of the strong phase difference ϕ, for $\phi_P = \pi/6$.

the phase space integration appearing in Eq. (13), since in our formalism $s = m_B^2$. Finally, one can argue that the effect of the higher states is simulated in a certain sense by the Goldberger-Treiman procedure, since it ensures a reality property of the discontinuity, which is normally valid when the unitarity sum is not truncated.

For simplicity, in the present application of the method we neglected the amplitudes suggested to be small by the quark diagrams, which introduces some model dependence in the final results. A more complete treatment including all the amplitudes is possible, by the simultaneous use of several dispersion relations for weak amplitudes correlated through rescattering effects.

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